Eigenanalysis of Multibody Systems

Joe Dinius
AME 553

May 11, 2010

Table of contents

Eigenanalysis Basics

- The basic idea behind eigenanalysis of multibody systems is to reduce a complicated system of the form
$$\begin{pmatrix} \mathbf{M} & -\mathbf{D}^T \\ \mathbf{D} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{v}} \\ \lambda \end{pmatrix} = \begin{pmatrix} \mathbf{g} \\ \gamma \end{pmatrix}.$$
to the more tractable linear system
$$\mathbf{M}\ddot{\delta} + \mathbf{K}\delta = \mathbf{0}. \tag{1}$$

- The matrix $\mathbf{K}$ is calculated as follows
$$K_{ij} = \frac{\partial \mathbf{F}_i}{\partial \delta_j}. \tag{2}$$
$\mathbf{F}$ is the vector of applied forces plus reaction forces.

Basics cont.

- The derived second order model is only valid near *fixed points*.
- Inverting both sides and rearranging the result yields the following

$$\ddot{\delta} = -\mathbf{M}^{-1}\mathbf{K}\delta. \tag{3}$$

  This equation is precisely the ODE for the *n*-dimensional simple harmonic oscillator (n is the rank of $\mathbf{M}^{-1}\mathbf{K}$). The eigenvalues of $\mathbf{M}^{-1}\mathbf{K}$ are the natural frequencies of the system.
- Knowledge of natural frequencies of multibody systems is important in
  1. System Design
  2. Control System Design
- Linear problems are easier to solve than nonlinear ones, so why not start there?

1D Spring

- The figure below shows the 1D spring system to consider

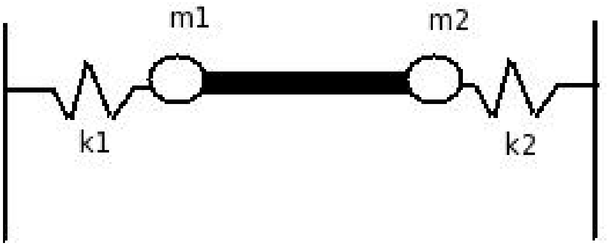


Figure: Diagram of 1D spring system

1D Spring cont.

- The equations of motion are

$$\begin{pmatrix} m_1 & 0 & 1 \\ 0 & m_2 & -1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{x}}_1 \\ \ddot{\mathbf{x}}_2 \\ \lambda \end{pmatrix} = \begin{pmatrix} k_1 \Delta x_1 \\ k_2 \Delta x_2 \\ 0 \end{pmatrix}.$$

- The natural frequency is

$$\omega = \sqrt{\frac{k1 + k2}{m1 + m2}}.$$

- The augmented $\mathbf{K}_*$ matrix is

$$\mathbf{K}_* = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

1D Spring cont.

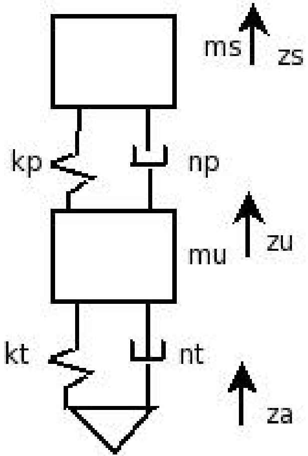
- Use numerical parameters

$$
\begin{aligned}
m_1 &= 3.1 \\
m_2 &= 1.9 \\
k_1 &= 14 \\
k_2 &= 10.7.
\end{aligned}
$$

- $\omega^2 = 4.94$ from above.
- Eigenvalues of $\mathbf{M}_*^{-1}\mathbf{K}_*$ are $\{4.94, 0\}$.

Quarter Car Diagram

- The linear quarter car diagram is seen in the figure below



Quarter Car Diagram

- The system has 2 translational DoF, 0 rotational DoF
- Drive the system with a 10Hz signal and watch frequency domain behavior.
- Use numerical parameters

$$\begin{aligned} m_s &= 241.5 \\ m_u &= 41.5 \\ m_a &= 50 \\ k_p &= 140000 \\ k_t &= 400000 \\ n_p &= 6000 \\ n_t &= 10000 \end{aligned}$$

Quarter Car

- The eigenvalues of $\mathbf{M}_*^{-1}\mathbf{K}_*$ do not represent the observed frequencies in this system. The values are close, but do not match as closely as the eigenvalues of $\mathbf{M}^{-1}\mathbf{K}$.
- The eigenvalues of $\mathbf{M}^{-1}\mathbf{K}$ are $\{9368.6, 579.7, 0\}$. These correspond to frequencies of 15.6 and 3.8Hz, respectively.

Quarter Car cont.

- Plots of time- and frequency-domain signals of the quarter car system are shown below

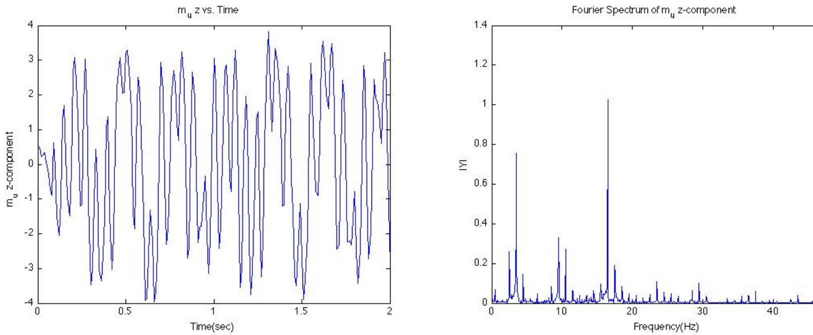


Figure: Quarter Car Signals. Left: Time-Domain and Right: Frequency-Domain

Conclusion

- Only $\mathbf{M}^{-1}\mathbf{K}$ should be used to calculate natural frequencies!
- Quarter car model frequency response matches expected output
- Questions?